Solutions to Exercises

Exercise 1: Elemental lead (Pb) has a superconducting transition at 7.2 K. A cuboid sample of Pb has dimensions (l × w × h) of (4 × 0.5 × 0.5) mm. At 7.5 K the voltage across two contacts (V^+ and V^-) was measured to be 3.52 nV when a current of 1 mA was applied across two other contacts (I^+ and I^-) as shown in Figure 3.

i. Work out the resistivity of the sample and compare it to the literature value of ρ = 25 nΩ.cm at 7.5 K. If your value is different, why do you think this is? (Hint: think about the geometry factor A/l in the diagram and where the contacts are placed on the sample).

Solution: *Using Ohm's law (**R** = V/I) we can obtain the resistance*:

R = 3.52 nV / 1 mA = $3.52 \times 10^{-9} / 1 \times 10^{-3} = 3.52 \times 10^{-6}$ Ω

R = 3.52 μΩ

*We can then use the given equation to work out the resistivity **ρ***:

$$\boldsymbol{\rho} = R\frac{A}{l} = (3.52\mu\Omega) \times \left(\frac{0.5mm \times 0.5mm}{4mm}\right)$$

$$\boldsymbol{\rho} = (3.52 \times 10^{-6}) \times (6.25 \times 10^{-5})\ \Omega\mathrm{m}$$

$$\boldsymbol{\rho} = 22\ \mathrm{n}\Omega.c\mathrm{m}$$

The values appear to differ by 3 nΩ.m (12 %), this could be due to the fact that the sample voltage contacts are not at the very ends of the sample and so the geometry factor is not correct (i.e the length seen by the measurement is not 4mm, but some number smaller than this). For this reason, it is important to measure where the contacts on a sample are placed during an experiment.

ii. The Pb sample is then cooled below 7.2 K and the voltage across V^+ and V^- is again measured for the same applied current of 1mA. The new voltage was recorded as 0.4 pV, what resistivity does this correspond to? What has happened to the sample of Pb?

Solution: *Using Ohm's law (**R** = V/I) we can obtain the resistance:*

R = 0.4 pV / 1 mA = 0.4 × 10^{-12} / 1 × 10^{-3} = 0.4 × 10^{-9} Ω

R = 0.4 nΩ

*We can then use the given equation to work out the resistivity **ρ***:

$$\boldsymbol{\rho} = R\frac{A}{l} = (0.4\text{n}\Omega) \times \left(\frac{0.5mm \times 0.5mm}{4mm}\right)$$
$$\boldsymbol{\rho} = (0.4 \times 10^{-9}) \times (6.25 \times 10^{-5})\, \Omega m$$
$$\boldsymbol{\rho} = 25 \text{ f}\Omega.\text{m}$$

This tiny value for the resistivity (femto = 10^{-15}) suggests that the sample has transitioned into the superconducting state and the measured value is just noise picked up by the measurement device.

iii. Why do you think it is important to use four contacts to measure the resistance and not two?

Solution: *Using four contacts allows a measurement of the sample only, without additional resistance coming from the measurement leads. If a two point measurement was used then the measured resistance would include the resistance of the measurement leads, this is very important when measuring very small resistances.*

Exercise 2: In 1962, Quinn and Ittner devised an experiment in which a current was sent around a 'squashed tube' made from two sheets of lead separated by a thin layer of silicon oxide. Elemental lead is a superconductor below 7.2 K and silicon oxide is an insulator. This is shown in Figure four.

The tubes inductance (L) was estimated at 1.5 × 10^{-13} Henrys. After 8 hours, there was no detectable change in the current (to within 1.5% precision i.e. the current was a minimum of 98.5% of its original value).

i. Estimate the maximum possible resistance of the tube for circulating currents.

Solution

Using Ohm's law, the maximum resistance R_{max} *occurs for the minimum in current* ***I,*** *using the information given in the question (*L *= 1.5 × 10*$^{-13}$*,* t *= 8 hours = 8 × 3600s = 2.88 × 10*^{4} *s) this occurs when:*

$$I(t) = 0.985 I_0$$

And so, using equation (2):

$$0.985 = exp^{\left(-R_{max}t/_L\right)}$$

Rearranging gives:

$$R_{max} = -\left(L/_t\right) ln(0.985)$$

Subbing in the given numbers for ***L*** *and* ***t*** *gives:*

$$R_{max} = 7.87 \times 10^{-20} \Omega$$

ii. How do you think Quinn and Ittner measured the current without disturbing the experiment? The dimensions of the tube used in the experiment are shown in Figure 4.

Solution

They had to measure the current indirectly without disrupting the experiment, this can be done by measuring the magnetic field produced by the current.

iii. Estimate the maximum possible resistivity ρ_{max} of the lead sheets. Compare your answer with the resistivity of pure lead at 300 K, which is ρ_{300} = 0.2 μΩ.m.

Solution

We know from equation (1) that the resistivity is proportional to the resistance and the geometry factor, the maximum resistance we have already worked out in part (i). The geometry factor is a bit more tricky, the length of the current path is indicated on the diagram by the red arrows and is approximately equal to:

$$l_{current} = 2 \times w_{SiO_2} = 2 \times 2.8\,mm = 5.6\,mm$$

Where $wSiO_2$ is the width of the silicon oxide layer.

The cross-sectional area A is equal to the length of the tube multiplied by the thickness of the Pb sheet:

$$A_{current} = 20\,mm \times (1.5 \times 10^{-6}\,mm) = 3 \times 10^{-8}\,m$$

And so:

$$\boldsymbol{\rho_{max}} = R_{max} {}^{A_{current}}/_{l_{current}} = (\,7.87\ \times 10^{-20}) \times \left({}^{3 \times 10^{-8}}/_{5.6 \times 10^{-3}}\right)$$

$$\boldsymbol{\rho_{max}} = 4.21 \times 10^{-25}\,\Omega m$$

Comparing this to the value at 300 K (ρ_{300} = 2 × *10*$^{-7}$ Ω.m) we see that there are 18 orders of magnitude between the room temperature value and the value below the superconducting transition. *Note that this is quite a rough calculation as the current would be expected to redistribute inside the sample, however our answer does serve as an upper limit for the resistivity of the sample.*